

The Grapes of Math

By

Andrew Breslin

It is impossible for a cube to be the sum of two cubes, a fourth power to be the sum of two fourth powers, or in general for any number that is a power greater than the second to be the sum of two like powers.

I have discovered a truly marvelous demonstration of this proposition that this margin is too narrow to contain.

- Pierre De Fermat

Found handwritten in his copy of Diophantus' *Arithmetica*
after his death in 1665

It remained unproven for 330 years.

Mathematics is beautiful, but get a bunch of mathematicians together and things can get positively ugly. Down at the Frank Roland Institute for Numerical and Spatial Studies, there was always a difference over differentials or a fracas over fractals. Gather nine powerful egos, capable of disagreeing on levels beyond most people's comprehension, and you can't expect them to form a softball team. The arguments were an intoxicating rush and I embraced them like a wino does a bottle of Mad Dog 20/20.

None of us had ever met the mysterious philanthropist Dr. Roland, but we sometimes wondered if he did not secretly monitor our grand intellectual battles, like a super-cerebral little boy who'd never quite outgrown the thrill of putting incompatible insects into a glass jar just to see what would happen.

It was a November morning some years ago. The 20th Century was winding down to its anticlimactic conclusion. Moods were sour, tensions high. The air smelled of mathematics, that is to say, just like a fresh pot of strong coffee. The late great itinerant genius Paul Erdős once defined mathematicians as machines that convert coffee into theorems, and none of us was willing to attempt a disproof by contradiction, dissuaded by the promise of splitting headaches that accompany caffeine withdrawal. Our cups filled and steaming, we gathered around the big oak table in the conference room. A few leaves from some of its luckier relatives clung to the outside of the large windows, adhered by the morning's light drizzle, weather that all too well mirrored our collective mood. We should have been triumphant. We should have been ecstatic. Instead the nine of us stared into our coffee cups and waited for someone to break the gloomy silence. We'd all shared a grim surreality ever since the proof was announced.

"First of all, this sure as hell isn't the proof Fermat had in mind. He didn't have all this high-falootin' modular-form business," Wally whined. "He just used good old fashioned number theory like any self-respecting 17th century mathematician would. Even if this proof holds water, it isn't elegant. It's not the 'marvelous little proof' we've all searched for."

Just about every mathematician I know and, statistics strongly suggest based on the evidence of this sample, the majority of the ones I haven't met, has at some point fantasized about proving Fermat's famous theorem. The small minority who have not have, with few exceptions, fantasized about disproving it.

It was never properly a theorem at all, but 'Fermat's last conjecture' just doesn't have the right ring to it. None of us could make a living trying to solve this puzzle, though. Even our eccentric benefactor, who pays nine moderately large salaries for abstract mathematics only rarely having any commercial application, was unwilling to finance that, so we'd been forced to tilt at other mathematical windmills.

Wally devotes his life to pi, and he will never run out of work. Billions of digits have already been determined, but there are trillions, quadrillions, quintillions waiting after that, and plenty of room to play around looking for patterns. He fancies himself a great explorer. The DeSoto of the mathematical world. He has an uncomfortable relationship with Dietrich who has no love for pi, but instead immerses himself in the beauty and mystery of e , the base of natural logarithms and the second most famous transcendental number, after pi. But e has always played Ben Johnson to pi's Shakespeare,

Gene Kelly to pi's Fred Astaire, and I suspect this gave Dietrich an inferiority complex, manifested, as they often are, in belligerence and fractiousness.

Dietrich and Wally are both slightly paunchy and balding. If not for Dietrich's thick German accent, it would be easy to mistake them for brothers, a conclusion bolstered by their almost constant bickering. We're usually careful not to invite them to the same social functions, for they can seldom share a room without coming to mental blows.

"Ha!" Dietrich spat. "You place far too much faiss in Fermat, Vally. Zat deceitful little Frenchman never even had a proof if you ask me. Zee sadist had nossing in mind but to torment generations of massematicians for centuries. Ze whole affair is a diabolical ruse zat he carefully orchestrated, and zen he craftily died before anyone could bring him to task on it. Now Euler, *zere* vas a massematician I could respect! None of zis sneaky writing in margins. If he had somesing to say, he'd write volumes on it!"

We paused to observe a moment of respect for the absurdly prolific Leonhard Euler, whom every one of us considered a demigod. It's worth noting that even Euler had tried and failed to prove Fermat's 'theorem,' but I said nothing to besmirch his divine reputation.

Richard, a tall and muscled Kenyan, is immersed in probability research. Being a playful and deviously creative bunch of madcaps, we folks at the institute nicknamed him 'Lucky.' Some mathematicians employ a technique called the 'Monte Carlo Method,' which uses large sets of random numbers to test certain ideas. Richard has invented what we at the institute call the 'Atlantic City Method,' whereby he goes to New Jersey on research projects and invariably comes back with less money and more theories.

“But it must be conceded, gentlemen, and lady,” he said, his voice deep and rich, sounding the way coffee tastes, “that Monsieur Fermat proposed a mathematical truth that hundreds of years and legions of mathematicians could not prove false. A single counter-example would have sufficed, but none was ever found, even utilizing modern mathematical theory, and modern technological tools of calculation. If he had no proof at all, why would he be so certain of the truth of the statement?”

“Intuition!” Melanie answered. “Many truths are self-evident but fiendishly slippery to prove.”

Melanie is married to another member of the institute, an undeserving number theorist named Rolf. Both are irritatingly attractive, evoking desire and envy, respectively, in their fellow mathematicians, or at least in me. They’re both so damn symmetrical. Scientists who study the evaluation of physical human beauty almost universally tout this as the single most significant factor. This resonates well with me. A balanced equation is far more beautiful than an inequality. And the lovely left half of Melanie’s beautiful body was perfectly balanced by her radiant right.

Her sheer comeliness notwithstanding, Melanie’s analytical prowess is irresistibly sexy. It’s difficult to strike up conversations in singles bars on the subject of quaternions, for example, and I can attest from personal experience that this is an excellent strategy for going home alone. But Melanie and I had such a discussion once, and it sent my hormones into an uproar. The way she would say the word “multiplicative,” it was all I could do not to seek a union of our respective sets right there on the spot.

For years I'd secretly desired her and wondered what she saw in Rolf, apart from his strong and perfectly symmetrical chin. He was a mediocre mathematician on a good day, and drunk on a bad one. And a mean drunk, to boot. He'd been grumpy and ill tempered for at least as long as I had known him, and I suspect since the day he was evicted from the womb.

"It was undoubtedly intuitively apparent to Fermat that the sum of two n th powers cannot itself be an n th power if n is greater than two," Melanie asserted. "Mathematical intuition always guides rigorous proof. If it didn't, computers would have made mathematicians obsolete a long time ago, and we all still have jobs."

Rolf and Melanie both work with primes, and much of their work involves improving methods for identifying new ones. Prime number generators are sort of the mathematical analogue to a perpetual motion machine, and in much the same vein, eventually run out of steam. But their work had identified many new large primes, which have recently become valuable commodities for use in encryption, a development we still haven't gotten used to. It's always a little shocking when someone finds a genuine use for number theory. Sometimes we feel a little dirty when our pure mathematics finds practical application. We can almost feel the admonishing eyes of professor Hardy.

Ollie, a tall and imperious Englishman, scoffed at these primes, as his numbers are much larger. He works with immense, sky-scrapingly large numbers, far larger than the largest known primes, which, had only a few hundred thousand digits back then and have only grown to a few million since. Ollie worked with googolplexes, megistons, mosers, and other such

entities denoting unfathomable numerical humongosity. He's even been known to stray into the transfinites, orders of infinity, speculation on which can drive the unprepared mind right over the edge and into intellectual oblivion. And as his numbers are enormous, so too is his ego. Were these to be quantified, Ollie's would require exponential notation.

"Well of *course* it's intuitively apparent to us denizens of the late 20th century," he snorted, his upper class British accent exaggerating his pretentious haughtiness. "It *has* been regarded as true for three centuries, but it wasn't obvious when Fermat wrote it. He must have had some solid basis for the assertion. After all, if it turned out to be false he'd look positively ridiculous posthumously, and what could possibly be more intolerable than that?"

Indeed, to a mathematician there are few prospects more horrifying. We don't expect to live lives of luxury and shocking hedonistic debauchery. To be plastered on the covers of entertainment magazines and be surrounded by beves of math groupies. Hope, perhaps, but certainly not expect. No, mathematical immortality, that's the prize. To be remembered after we die, leave a lasting legacy and forever change mathematics. Some day, after we are long gone, we want some student somewhere to read our preserved thoughts and think, "Wow! What a great mathematician!"

To be remembered as a presumptuous and arrogant boaster who made an assertion that turned out to be wrong would be as perfect a hell as any the most nefarious of demons could devise.

Morty, normally quite shy, managed to share a few ideas.

"M-m-maybe he just made an error," he said, pausing to push up his downward-sliding spectacles. "Maybe he thought he had a proof, but it was flawed. H-h-he may have divided by zero or something."

Morty is perhaps the strangest member of our little group. (Morty might say "divergent element of our finite set.") He's short and wears thick, heavy glasses that he constantly pushes back along his nose to counter the relentless efforts of gravity. While most of us are, in contrast to popular misconception, quite well-adjusted socially, and conversant in a wide range of subjects far removed from mathematics, Morty never quite got a handle on the rest of the universe outside of mathematics. I am pretty sure that if Melanie were to say "multiplicative" to him in that sexy way she has, it would probably kill him on the spot.

Imaginary numbers, square roots of negatives, are his demesne and we often needle him about this. "Get some *real* numbers!" we tease, though Morty is never amused. He wears a T-shirt with a lowercase *i* on it to display his great passion to the world. The square root of -1, *i* forms the cornerstone of imaginary and complex numbers. Morty's shirt-borne message is lost on the masses who probably presume the *i* is an ironic statement about egomania combined with a twist of humility. Some might observe that all of us are a bit out of touch with the real world, but Morty is definitely the worst. Even his calculator doesn't understand him.

Dividing by zero is, of course, mathematically blasphemous and grotesque. When any of us realize that we've inadvertently made such a reprehensible error, we immediately bend to one knee and appeal to the gods of mathematics for forgiveness, invoking the sacred names of Pythagoras,

Euclid, and Archimedes. Not really, but that gives an idea of the seriousness of the offense.

It is doubtful that the great Fermat divided by zero, though it is easier to do than it sounds. A term could be set to zero, and then one could divide by that term to cancel something out, not realizing that the term is equal to zero. Good mathematicians do it all the time, and many of them will eagerly show their scars and recount the horror stories of having made this dreadful mistake. Still, Fermat probably didn't, but he could have made some kind of error. Although acknowledged as brilliant, he was equally noted for being sloppy.

Rolf sided with Morty, "I'm sure that French braggart made a mistake. Fermat was undisciplined and messy. What ever possessed him to undertake mathematics as a hobby? I don't go around making legal decisions for *my* amusement!"

Fermat was not a professional mathematician; he was a jurist. Mathematics was simply an entertaining diversion for him, thus earning him the nickname "prince of the amateurs." Rolf's hostility was a blatant advertisement of his resentment of Fermat's genius, for the French legal scholar had accomplished much more with his "hobby" than Rolf ever would with his highly specialized career, for which he was handsomely paid. Ever tactful, I ignored the opportunity to point this out.

I offered what insight I had:

"It seems then, that there are a limited number of logical possibilities.

To wit:

“One: Fermat made some sort of mistake, and never had a consistent proof, though he thought he had one.

“Two: Fermat suspected that the assertion was true, guided by his intuition, but found a proof to be all but impossible, and so proposed to have discovered one out of a malicious desire to torture his intellectual descendants, or to secure his immortality, succeeding wildly in either case.

“Three: Fermat secretly developed entire branches of mathematics that required the life’s work of many brilliant mathematicians over the course of centuries to rediscover, but kept all the results of this staggering amount of effort a secret except for one obscure little conundrum. Or four . . .”

Here I paused dramatically, taking in the circle of eyes with my own in a grand sweep.

“Fermat discovered a simple, elegant, whole and consistent proof using nothing but the tools of 17th century mathematics.”

“I think we can strike number three as exceedingly unlikely,” Terry observed.

Terry is interested in topology. Sometimes he spends hours staring at and turning his coffee cup, and there are few who could do this and honestly say that they were working. Often we observe a vacant look in Terry’s eyes while engaged in conversation with him and get the disturbing impression that he is appraising the topological characteristics of our bodies.

He would be quick to point out that regardless of individual girth or stature, most human bodies are “topologically equivalent.” This is a somewhat unsettling notion. Terry himself weighs in at 265 pounds, and frankly looks quite a lot like an enormous egg. I’ve never said anything about

this, though some of the others refer to him as “Humpty Dumpty” when he’s not around.

Everyone nodded at Terry’s proposition. The ovoid mathematician paused to bite a doughnut. These were his favorite foods, classic representatives of the torus manifold about which topologists are endlessly excited. He swallowed quickly and continued, a sugar mustache now resting on his upper lip.

“And despite the wishes and desires of the romantic within each of us, number four seems less and less likely with the passing of each century.”

Again we nodded, this time sadly, for it seemed undeniable that the simple and elegant proof of Pierre de Fermat was to be classed with the pegasi and unicorns, a fabled, fantastic and beautiful beast, which was, alas, unreal.

“Which brings us to this Wiles fellow and his proof. What of it?” Ollie asked, folding his arms with a smug shrug.

“A hack,” Dietrich opined.

“Inspired nonsense,” Wally added, in a rare display of apparent agreement with Dietrich.

“It seems unlikely that it will stand the test of time,” Richard offered, somewhat more diplomatically.

Nobody was making any eye contact with anyone else. The reason was clear to me: Everyone was spouting a steady stream of undiluted bullshit.

“Why?” I asked “Why are you all so certain? Have you read the proof? Did you understand all of it? Did you find any flaws you can point to?”

"Just one," Rolf said.

"What's that?" I asked

"It wasn't my proof."

At last, Rolf had cut to the heart of the matter. Pure spiteful intellectual envy.

"And what about the Shimura-Taniyama conjecture?" Ollie asked, "Surely they shall now bathe in glory."

"Taniyama killed himself forty years ago," I noted.

"That's irrelevant," Ollie replied with a dismissive wave and a snort.

The work of Shimura and Taniyama, pairing modular forms and elliptic curves, had formed the basis of Wiles' proof. Indeed, they had already bathed in a steep tub of glory, for their ideas had been immensely influential to modern mathematics, long before anyone got the idea to apply them to Fermat's last theorem. But Taniyama never had a chance to enjoy any of it, having taken his life long before his labors had borne so much fruit. We'll never know if his suicide was related to frustration at his inability to prove his own conjecture, or Fermat's last theorem (also a conjecture) for that matter. But his life and death teach us a valuable lesson: after you die, some of your mathematical scribbling may prove more useful than you had ever dreamed. Die happy!

"Can't you all just be satisfied that a proof has been found?" Melanie admonished. "Can't you just appreciate its beauty? Isn't it good enough that the world has been enriched by the discovery? What if medical researchers approached diseases the same way you approach mathematical proofs,

dreading that someone else would find a cure first, and becoming enraged and depressed when they did?"

Melanie was obviously not well acquainted with any medical researchers. I let the point slide.

We squirmed uncomfortably. It was true. We were a bunch of conceited brats when it came right down to it. Very intelligent conceited brats, yes. But brats, nevertheless. We couldn't stand to see another get the grapes. Sour, sour, oh those grapes are so so sour.

I broke the silence that had again settled gloomily over all of us.

"I always thought Fermat's last theorem was overrated, anyway. Now the Goldbach Conjecture, *that's* the last great mathematical puzzle. That's a real challenge. If we set to work on that, it would truly assure us of immortality," I suggested.

Everyone heartily agreed. We spent the rest of the day exchanging ideas on the Goldbach Conjecture and let Pierre de Fermat rest in peace.

THE END

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